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An Automatic Microwave Phase Comparator[*]

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Summary—**A method for passively measuring the phase angle between two signals of the same frequency is described. While simple in concept, the system has no ambiguities throughout 360° and is independent of relative signal amplitudes because phase angle is displayed orthogonally to amplitude. Consisting principally of two hybrids with detectors and an X-Y indicator, the system contains no moving parts or active phasing devices. In addition to making routine phase measurements, it can be readily applied to automatic direction finders, polarization analyzers, and impedance plotters.**

INTRODUCTION

A SIMPLE method for measuring the relative phase angle between two signals of the same frequency is presented. It differs from previous phase-measuring techniques in that it indicates phase angle automatically and instantaneously without using modulators[1,2,3] or frequency translators.[4,5] The principle of phase comparison described presents no ambiguities throughout 360° and is independent of relative signal amplitudes.[6] It is classed as a passive system since there are no moving parts or active phasing devices. The measuring speed is limited only by the indicating device.

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The RF components of the phase comparator circuit described comprise standard microwave components, namely two 3-db hybrids, two two-way power dividers, two pairs of detectors, and interconnecting transmission lines. A suitable display device is also required. Each of the two signals whose phase difference is to be measured is divided into two equal parts and thereafter combined in the hybrids. A pair of detectors is associated with each hybrid. By treating the difference of the detected outputs of one pair of detectors orthogonally to the difference of the detected outputs of the other pair of detectors, a single-valued function of phase for 360° is obtained.

THEORY

Consider the circuit of Fig. 1 with input signals whose amplitudes are proportional to A and B and have a relative phase angle of ϕ degrees between them. Hybrid I is a ring network (180° hybrid) connected to the power dividers by two equal lengths of transmission line. Hybrid II is another ring network which is connected to the power dividers through two lines differing in length by one-quarter wavelength (equivalent to a 90° hybrid). The complex signals at output terminals a and b of hybrid I are proportional to

$$E_a \approx Ae^{+j(\varphi/2)} - Be^{-j(\varphi/2)} = (A - B)\cos\frac{\varphi}{2} + j(A + B)\sin\frac{\varphi}{2} \qquad (1)$$

$$E_b \approx Ae^{+j(\varphi/2)} + Be^{-j(\varphi/2)} = (A + B)\cos\frac{\varphi}{2} + j(A - B)\sin\frac{\varphi}{2}\cdot \qquad (2)$$

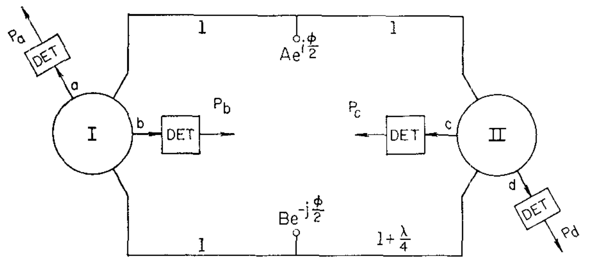

Fig. 1—Phase comparator circuit.

If the detected amplitudes are proportional to $|E|^2$ (identical square law detectors and ideal circuit components operating within their intended frequency range)

$$P_a \approx |E_a|^2 \approx (A^2 + B^2) - 2AB\cos\varphi \qquad (3)$$

$$P_b \approx |E_b|^2 \approx (A^2 + B^2) + 2AB\cos\varphi. \qquad (4)$$

The difference of the detected amplitudes is

$$P_b - P_a = 4AB\cos\phi. \qquad (5)$$

Similarly, the detected amplitudes at output terminals c and d of hybrid II are

$$P_c \approx |E_c|^2 \approx (A^2 + B^2) + 2AB\sin\varphi \qquad (6)$$

$$P_d \approx |E_d|^2 \approx (A^2 + B^2) - 2AB\sin\varphi. \qquad (7)$$

The difference of these detected amplitudes is

$$P_c - P_d = 4AB\sin\phi. \qquad (8)$$

Displaying these difference amplitudes orthogonally, *i.e.*, $P_b - P_a$ along the X axis and $P_c - P_d$ along the Y axis, the total indicated power is

$$P_t \approx 4AB\cos\phi + j4AB\sin\phi = 4ABe^{j\phi}. \qquad (9)$$

This function is single valued over 360° and has an amplitude proportional to the product of the signal amplitudes. Further, the amplitude is displayed as a radius to the circle described by φ.

It may be noted that the sum of the detected signals is a quantity which is independent of the relative phase and proportional to the total input power.

EXPERIMENTAL RESULTS

Using the circuit shown in Fig. 2 as a source of constant amplitude signals—one of fixed phase and one of variable phase—the detected amplitudes at the output terminals a, b, c and d of a printed microstrip version of Fig. 1 were measured with a bolometer detector. These measured outputs as a function of the relative phase of the two input signals are shown in Fig. 3 as plotted points. A theoretical power curve is fitted over the measured values.

Fig. 4 is a display obtained using four crystal detectors when $P_b - P_a$ is applied to the X axis and $P_c - P_d$ is applied to the Y axis of an $X-Y$ recorder. That is, the outputs of the detectors attached to hybrid I are connected such that only the difference of the detected signals is developed across a load (the X axis of the recorder). The detectors attached to hybrid II are connected in a similar manner to the Y axis. The radials approximately every 45° were obtained by stopping the probe of the slotted line at intervals of 45 electrical degrees and allowing one of the signals to go to zero. The various circles are labeled as to the difference in signal amplitudes.

The circles drawn by the X-Y recorder are not perfect, resulting in a phase error reading of as much as approximately ±4°. The VSWR at either signal input terminal with matched loads on the four output terminals was below 1.1 over at least an 8 per cent frequency band. Also, the power division between the four detector arms was equal to within 0.1 db, from either input terminal. The precise difference in length between the two transmission lines leading to hybrid II was not known. A differential line length of $\lambda/4$, which was in accordance with previously measured

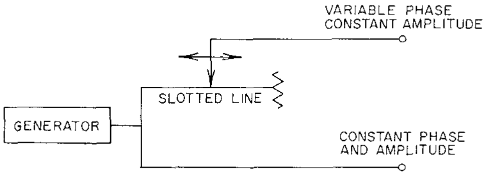

Fig. 2—Test signals source.

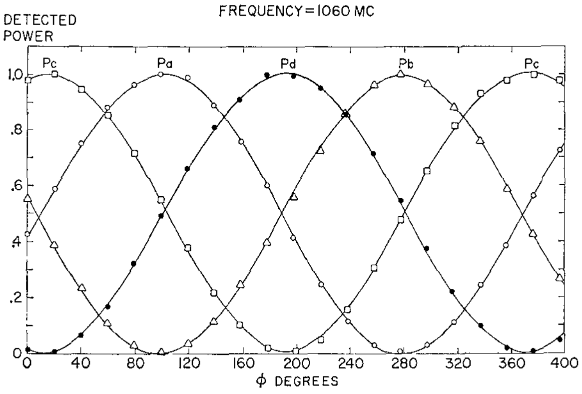

Fig. 3—Phase comparator output.

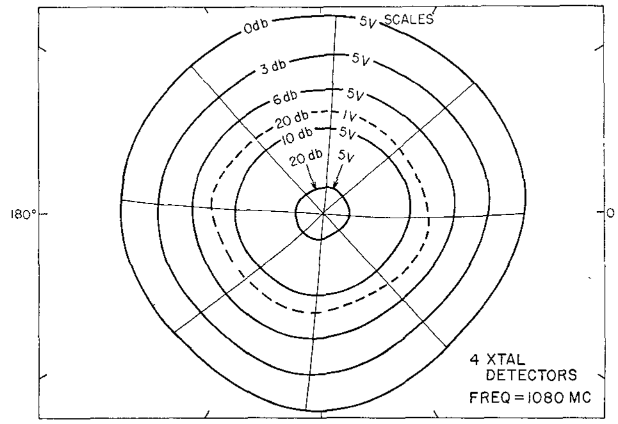

Fig. 4—Phase comparator output displayed on X-Y recorder.

microstrip propagation constant,[7] was selected. The crystals used as detectors were not selected nor were their input impedances matched. This is believed to be the largest source of error appearing in Fig. 4. Bolometer detectors were not used because biasing is not provided at the X-Y recorder input terminals.

When the input signals are in-phase, or anti-phase, and when using ideal system components, the quadrature detected signals are equal in amplitude, making their difference equal to zero. This produces an X axis crossing. When the input signals are in phase quadrature, the detected outputs from the 180° hybrid are equal, producing a Y axis crossing.

The phase comparator described above has a relatively narrow frequency band because of the fixed quarter-wavelength phase shifter that is employed. A broad-band version would use a 90° 3 db hybrid in place of the quarter-wavelength phase shifter and the associated 180° hybrid.

APPLICATIONS

This phase comparator can be used for routine phase measurements with or without accompanying change in attenuation with change in phase. Thus, the comparator will permit determination of change in attenuation with change in phase, or of change in phase with change in attenuation, because these two functions are displayed orthogonal to one another.

It may be noted that a phase difference of $\phi = n\alpha$ between the input signals produces an identical variation of $\phi = n\alpha$ in the output power indication. For example, when measuring the phase difference between currents induced by a signal on two antennas separated by five wavelengths, the phase comparator output undergoes ten complete cycles for a change in signal direction, in a plane containing the antennas, of $\phi = 180°$. An X-Y indicator would trace out ten complete circles for the 180° change in signal direction.

Similarly, this phase comparator can, with an addition to the input circuit, be used as an automatic impedance plotter, indicating impedance directly on a Smith chart. The addition to the input circuit could be simply a 180° hybrid. The required $\phi = 2\alpha$ variation *i.e.*, repetition of impedance every 180°, is obtained by virtue of comparing signals comprising incident and reflected waves.

When used in conjunction with an antenna array of oppositely sensed circularly polarized radiators, the phase comparator becomes an integral part of an automatic polarization analyzer. When used with a spiral antenna operating in the first two radiation modes simultaneously, it becomes part of an automatic direction finder with no moving parts or directional ambiguities over a hemisphere.[8,9]

Substituting $\phi = \Delta f$, where Δf is an incremental change in frequency, the phase comparator becomes a "Direction Sensitive Doppler Service" described by Kalmus.[10]

The phase comparator described, while quite simple in concept, is capable of unambiguous indication of phase angle over 360° and will independently indicate change in attenuation. The orthogonal treatment of phase and amplitude allows application to a wide variety of automatic devices.

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